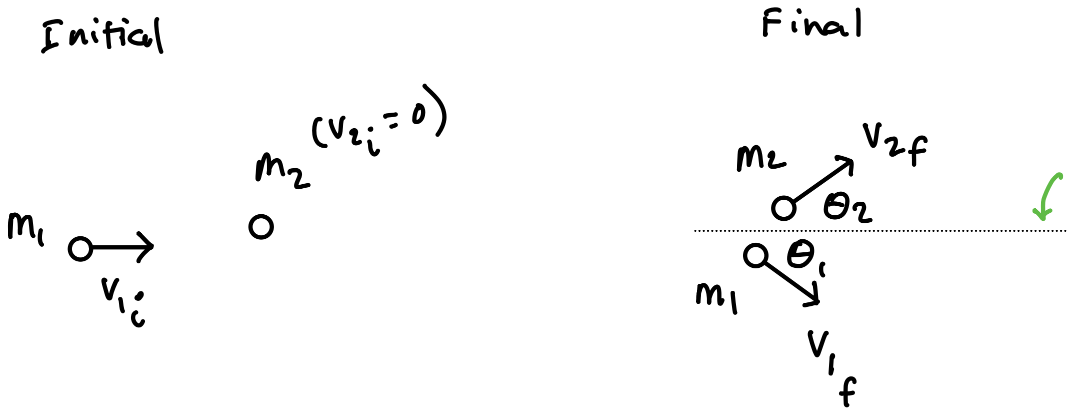
VECTOR ANALYSIS

* If dealing w/ 4 or more vectors, no real choice. ← algebra only
USE COMPONENTS. $(\hat{i}, \hat{j}, \hat{k})$

* If 3 vectors, you can do algebra ← $\hat{i}\,\hat{j}$ or geometry ← draw $\triangle$

$\vec{a} + \vec{b} = \vec{c}$ or $\vec{a} + \vec{b} + \vec{c} = 0$

Let's do a 2-D collision as an example.



← the original direction of m_1

So $\theta_i = 0°$

The Big Idea! → $\boxed{\Sigma \vec{P}_i = \Sigma \vec{P}_f}$ ← Conservation of Momentum!

$$\text{So} \quad \vec{P}_{1_i} + \cancel{\vec{P}_{2_i}} = \vec{P}_{1_f} + \vec{P}_{2_f}$$

$$\text{so} \quad m_1\vec{v}_{1_i} + \cancel{m_2\vec{v}_{2_i}} = m_1\vec{v}_{1_f} + m_2\vec{v}_{2_f}$$

↑ $v_{2_i} = 0$!

Algebra

$$m_1 v_{1_i}\hat{i} = m_1 v_{1_f}(\cos\theta_1 \hat{i} - \sin\theta_1 \hat{j})^{*} \leftarrow (\theta_1 = +)$$

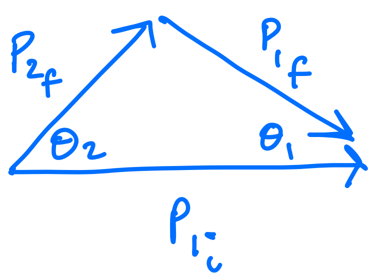
$$+ m_2 v_{2_f}(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j})$$

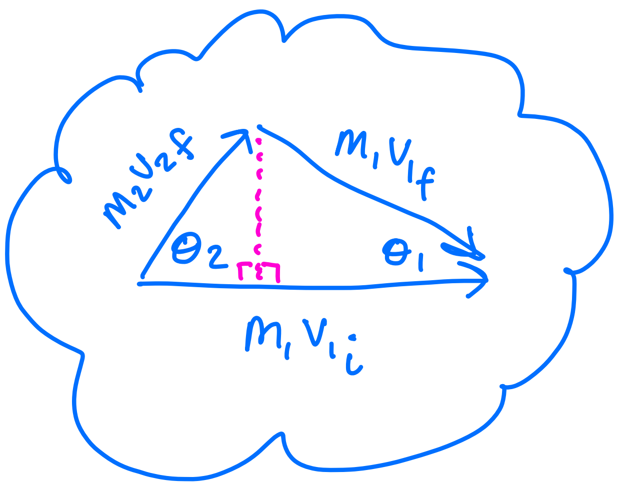
* or could have said "$m_1 v_{1_f}(\cos\theta_1 \hat{i} + \sin\theta_1 \hat{j})$" where θ_1 is negative

$$\boxed{\begin{aligned} \hat{i})\quad & m_1 v_{1_i} = m_1 v_{1_f}\cos\theta_1 + m_2 v_{2_f}\cos\theta_2 \\ \hat{j})\quad & 0 = -m_1 v_{1_f}\sin\theta_1 + m_2 v_{2_f}\sin\theta_2 \end{aligned}}$$

Geometry

$$\vec{p}_{1_i} + 0 = \vec{p}_{1_f} + \vec{p}_{2_f}$$





So drop an altitude (shown in pink) to make two smaller triangles. The heights of the two little triangles are the same, so we can say

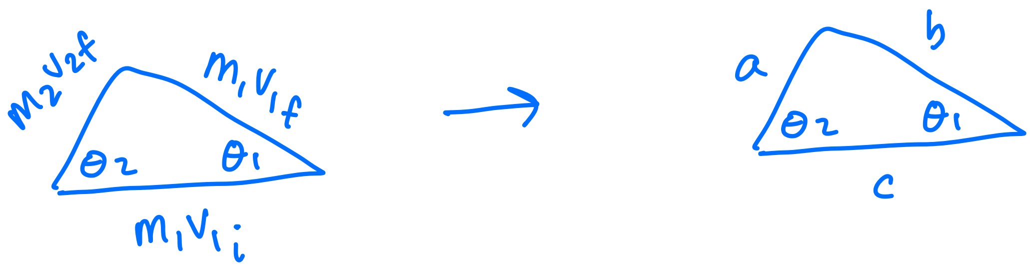
$$\boxed{m_2 v_{2f} \sin\theta_2 = m_1 v_{1f} \sin\theta_1}$$

And the bases of the two little triangles have to add up to the original, so we can say

$$\boxed{m_1 v_{1i} = m_2 v_{2f} \cos\theta_2 + m_1 v_{1f} \cos\theta_1}$$

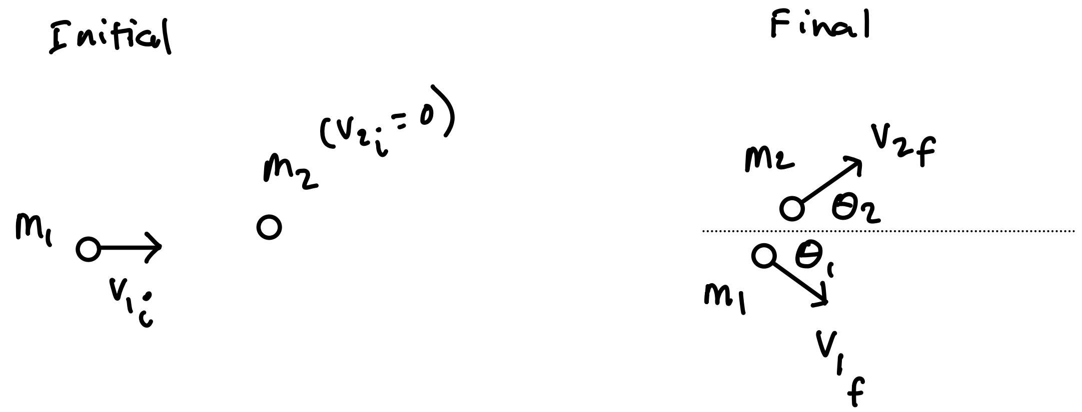
HEY! Those are the same equations as from the "algebra" way!

Really, you just have two ways to set up the equations. There is one advantage to the geometry setup though:



You have a triangle – so you can also use the Law of Cosines and Law of Sines – which gives you other equations to use.

Example:



A 2 kg object is moving with an initial speed of v_{1_i}. It collides off a 3 kg object which is initially at rest. The 3 kg object has a final velocity of 4.5 m/s at an angle of 35° (as shown in the diagram.) The 2 kg object has a final speed of 7 m/s. What was the initial speed of the 2 kg object?

① $\sum \vec{P}_i = \sum \vec{P}_f$ b/c momentum has to be conserved.

$$\therefore \quad (2)v_{1_i}\,\hat{i} = \underbrace{\left[(2)(7)\cos\theta_1\,\hat{i} - (2)(7)\sin\theta_1\,\hat{j}\right]}_{\vec{P}_{1_f}} + \underbrace{\left[(3)(4.5)(\cos 35)\,\hat{i} + (3)(4.5)(\sin 35)\,\hat{j}\right]}_{\vec{P}_{2_f}}$$

Let's separate out the two components (& do a little calculator):

$\hat{i})$ $\quad 2v_{1_i} = 14\cos\theta_1 + 13.5\cos 35$

$\hat{j})$ $\quad 0 = -14\sin\theta_1 + 13.5\sin 35$

So $\quad \sin\theta_1 = \dfrac{13.5\sin 35}{14}$

$$\boxed{\theta_1 = 33.6^\circ}$$

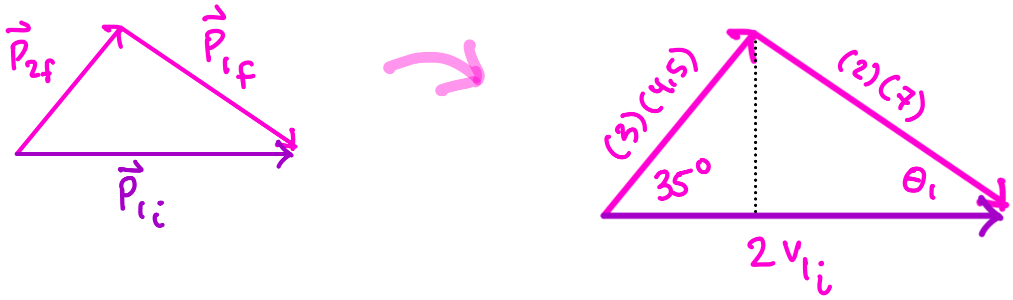
And so

$$2v_{1_i} = 14\cos 33.6 + 13.5\cos 35$$

$$\boxed{v_i = 11.4\ \text{m/s}}$$

(2) $\Sigma \vec{P}_i = \Sigma \vec{P}_f$ b/c momentum has to be conserved.

$\therefore$



$\therefore$ $(3)(4.5)\sin 35 = (2)(7)\sin\theta_1$

So $\sin\theta_1 = \dfrac{(3)(4.5)\sin 35}{(2)(7)}$

$$\boxed{\theta_1 = 33.6°}$$

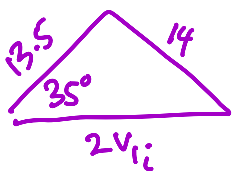
And so

$$2v_{1_i} = (3)(4.5)(\cos 35) + (2)(7)(\cos 33.6)$$

$$\boxed{v_{1_i} = 11.4 \text{ m/s}}$$

NOTE:

You could also have used the Law of Cosines from the triangle that was drawn:



$$14^2 = (13.5)^2 + (2v_{1_i})^2 - 2(13.5)(2v_{1_i})\cos 35$$

$$196 = 182.25 + 4v_{1_i}^2 - 44.23\, v_i$$

$$0 = 4v_{1_i}^2 - 44.23\, v_i - 13.75$$

So $v_{1_i} = \dfrac{44.23 \pm \sqrt{(44.23)^2 - 4(4)(-13.75)}}{2(4)}$

$$v_{1_i} = \frac{44.23 \pm 46.66}{8}$$

So $\boxed{v_{1_i} = 11.4 \text{ m/s}}$ (take the + root)